Isozaki, Y.
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A CLUSTER OF SETS OF EXCEPTIONAL TIMES OF LINEAR BROWNIAN MOTION

YASUKI ISOZAKI

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1. Introduction and the main theorems

Aspandiiarov-Le Gall [1] studied the following random closed sets K^-, K and K': Let $(B_t; t \geq 0)$ be a linear standard Brownian motion starting at 0, and let

$$\begin{aligned} K^- &= \left\{ t \in [0,1]; \int_s^t (B_u - B_t)du \leq 0 \quad \text{for every } s \in [0,t). \right\}, \\ K &= \left\{ t \in K^-; \int_t^s (B_u - B_t)du \leq 0 \quad \text{for every } s \in (t,1]. \right\}, \\ K' &= \left\{ t \in K^-; \int_t^s (B_u - B_t)du \geq 0 \quad \text{for every } s \in (t,1]. \right\}. \end{aligned}$$

They computed the Hausdorff dimension of K^-, K and K'.

Theorem ([1]). *It holds* $\dim K^- = 3/4$, $\dim K = 1/2$ *and* $\dim K' \leq 1/2$ *almost surely. The set* K' *is possibly empty or* $\dim K' = 1/2$, *both with positive probability. The same statements hold if the weak inequalities in the definition of* K^-, K *and* K' *are replaced by the strict inequalities.*

In this paper, we consider a cluster of random sets having various dimension.

For $\alpha \geq 0$ and $c > 0$, we define the following functions $V(\alpha, c)$ increasing on $\mathbb{R}$:

$$V(\alpha, c; y) = y^\alpha \quad \text{for } y > 0; V(\alpha, c; 0) = 0; V(\alpha, c; y) = -\frac{|y|^\alpha}{c} \quad \text{for } y < 0.$$

Let α, α_+, $\alpha_- \geq 0$, c, c_+, $c_- > 0$ and write V for $V(\alpha, c)$, $V_\pm$ for $V(\alpha_\pm, c_\pm)$. We define the random sets depending on the functions V, V_+ and V_-:

$$K^-(V) = \left\{ t \in [0,1]; \int_s^t V(B_u - B_t)du \leq 0 \quad \text{for every } s \in [0,t). \right\}, \tag{1.1}$$

$$K(V_-; V_+) = \left\{ t \in K^-(V_-); \int_t^s V_+(B_u - B_t)du \leq 0 \quad \text{for every } s \in (t,1]. \right\}, \tag{1.2}$$

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