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ON THE *n*-COMPLETENESS OF COVERINGS OF PROPER FAMILIES OF ANALYTIC SPACES

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0. Introduction

In this paper we investigate complex analytic completeness of certain unramified covers of proper families of analytic spaces with n-dimensional fibers. When $n = 1$, T. Ohsawa has studied the stability of unramified covering spaces of complex analytic families of Riemann surfaces, and proved the following ([13], [14]):

Theorem O. (1) *Let X be a connected complex manifold of dimension* 2 *and T the unit disk of* **C**. *Let $\pi : X \longrightarrow T$ be a proper surjective holomorphic submersion. Then every unramified covering space of X is holomorphically convex.* (2) *Let T be any contractible complex space, and X a complex space. Let $\pi : X \longrightarrow T$ be a proper surjective holomorphic map with one-dimensional fibers, and $\sigma : \widetilde{X} \longrightarrow X$ an unramified cover. Then a point $z \in T$ has an open neighborhood U such that $(\pi \circ \sigma)^{-1}(U)$ is holomorphically convex if and only if $(\pi \circ \sigma)^{-1}(z)$ is holomorphically convex.*

In connection with Theorem O, the author ([10]) and M. Coltoiu and V. Vâjâitu ([4]) have investigated completeness of the covering spaces of proper families with higher dimensional fibers. Here we shall prove a new result in this direction.

Let $\pi : X \longrightarrow T$ be a proper surjective holomorphic map of connected complex manifolds, and $n = \dim X - \dim T$ the relative dimension. Let $\sigma : \widetilde{X} \longrightarrow X$ be an unramified cover. We remark that when A is an analytic subset, $\pi^{-1}(A)$ and $(\pi\circ\sigma)^{-1}(A)$ have possibly non-reduced structures. Then we prove the following.

Theorem. *Let z be a point of T satisfying the following two conditions*: (i) $\pi^{-1}(z)$ *is a reduced connected complex space of dimension n*, (ii) $(\pi \circ \sigma)^{-1}(z)$ *has no compact irreducible component of dimension n, where $n = \dim X - \dim T$ is the relative dimension. Then there exists an open neighborhood U of z such that $(\pi\circ\sigma)^{-1}(U)$ is n-complete.*

It is well known that every n-dimensional reduced paracompact complex space is n-complete if it has no compact irreducible component of dimension n ([12], [6]). Our theorem is a relative version of this fact. We also remark that Coltoiu and Vâjâitu have