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ESTIMATES OF HITTING PROBABILITIES FOR A 1-DIMENSIONAL REINFORCED RANDOM WALK

MASAKI TAKESHIMA

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1. Introduction

In this paper, we will discuss the recurrence of a matrix type reinforced random walk $\overrightarrow{X} = \{X_n\}_{n\geq 0}$, with initial weights $\{w(0, j)\}_{j\in\mathbf{Z}}$ and a reinforcing matrix $A = \{a(n, j)\}_{n\in\mathbf{N},\, j\in\mathbf{Z}}$ of non-negative numbers. The transition mechanism of this walk is defined through its weight process $\overrightarrow{W} = \{w(n, j)\}_{n\geq 0,\, j\in\mathbf{Z}}$ in the following manner.

$$
\begin{aligned}
(1.1)\qquad P[X_{n+1} = j+1 \mid X_n = j,\ \{w(n,i)\}_{i\in\mathbf{Z}}] &= \frac{w(n, j)}{w(n, j-1) + w(n, j)},\\
P[X_{n+1} = j-1 \mid X_n = j,\ \{w(n,i)\}_{i\in\mathbf{Z}}] &= \frac{w(n, j-1)}{w(n, j-1) + w(n, j)}.
\end{aligned}
$$

The weight process $\overrightarrow{W}$ is a family of additive functional of $\overrightarrow{X}$, which are defined in terms of A.

$$
(1.2)\qquad w(n, j) = w(0, j) + \sum_{l=1}^{\phi(n,j)} a(l, j),
$$

where $\phi(n, j)$ is the total number that $\overrightarrow{X}$ crosses the edge $\{j,\ j+1\}$ up to time n;

$$
(1.3)\qquad \phi(n, j) = \sum_{l=1}^{n} 1_{\{\{X_{l-1},\, X_l\}=\{j,\, j+1\}\}}
$$

where 1_A is an indicator function of a set A. Throughout this paper we call the pair $[\overrightarrow{X}, \overrightarrow{W}]$ simply by a reinforced random walk. We shall abbreviate a reinforced random walk to RRW.

We call a path $\overrightarrow{X}$ recurrent if for every $j \in \mathbf{Z}$, X_n visits j infinitely often, and transient if for every $j \in \mathbf{Z}$, X_n visits j only finitely many times. If there exist $\alpha < \beta$ such that $\alpha \leq X_n \leq \beta$ for all n, then we say that the path $\overrightarrow{X}$ has finite range. We will introduce convergence tests of the a following function. Let $\Phi : (0, \infty)^{\mathbf{N}\cup\{0\}} \to (\,0,\ \infty\,]$ be given by