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DIFFEOMORPHIC EXTENSION OF BIHOLOMORPHIC MAPPINGS WITH SMOOTH MODULUS

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1. Introduction

Fefferman proved in [8] that any biholomorphic mapping between two smooth bounded strictly pseudoconvex domains D_1 and D_2 in $\boldsymbol{C}^n$ extends to a diffeomorphism of $\bar{D}_1$ onto $\bar{D}_2$. Later Fefferman's theorem was extended by Bell and Ligocka [7] and Bell [2].

Let D be a smooth bounded pseudoconvex domain in $\boldsymbol{C}^n$. Let $L^2(D)$ be the space of square-integrable functions on D. We denote by $H(D)$ the space of square-integrable holomorphic functions on D. The Bergman projection P is the orthogonal projection from $L^2(D)$ to $H(D)$. The domain D is said to satisfy condition R if P maps $C^\infty(\bar{D})$ continuously into $C^\infty(\bar{D})$. Bell's result [2] is as follows:

Let D_1 and D_2 be smooth bounded pseudoconvex domains in $\boldsymbol{C}^n$. If either D_1 or D_2 satisfies condition R, then any biholomorphic mapping between D_1 and D_2 extends to a diffeomorphism of $\bar{D}_1$ onto $\bar{D}_2$.

It is not known that any biholomorphic mapping between smooth bounded weakly pseudoconvex domains in $\boldsymbol{C}^n$ can be extended to a diffeomorphism onto the bouundary. Fornaess proved in [9] that any biholomorphic mapping $f\colon D_1 \to D_2$ between bounded pseudoconvex domains D_1 and D_2 in $\boldsymbol{C}^n$ with C^2-boundary extends to a C^2-diffeomorphism of $\bar{D}_1$ onto $\bar{D}_2$, if f has a C^2-extension $\hat{f}\colon \bar{D}_1 \to \bar{D}_2$. In this paper we shall prove the theorem of this type. Let D_1 and D_2 be smooth bounded pseudoconvex domains in $\boldsymbol{C}^n$. Using Bell's method we shall prove that any biholomorphic mapping $f\colon D_1 \to D_2$ extends to a C^∞-diffeomorphism of $\bar{D}_1$ onto $\bar{D}_2$, whenever $|f|^2$ is C^∞.

2. Preliminaries

Let D be a smooth bounded pseudoconvex domiain in $\boldsymbol{C}^n$. We denote by $W^s(D)$ the usual Sobolev space for $s>0$. A negaitve Sobolev space $W^{-s}(D)$ is the dual space of $W_0^s(D)$, where $W_0^s(D)$ is the closure of $C_0^\infty(D)$ in $W^s(D)$. We now consider the dual space $W^s(D)^*$ of $W^s(D)$ for $s>0$.

Let $\langle\ ,\ \rangle$ be the $L^2(D)$ inner product. For any $f \in L^2(D)$, $\langle \cdot, f\rangle$ is a