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ON ALGEBRAS OF 2-CYCLIC REPRESENTATION TYPE

Dedicated to Professor K. Shoda on his sixtieth birthday

By

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§1. Let A be an associative algebra with a unit and of finite dimension over an algebraically closed field K and $A=\sum_i\sum_j Ae_{ij}$ be a decomposition of A into a direct sum of directly indecomposable left ideals where $Ae_{\kappa,i}\cong Ae_{\kappa,1}=Ae_\kappa$ and let N be its radical.

Now if an A-left module (or an A-right module) $\mathfrak{m}$ is a homomorphic image of one of Ae_i (or e_jA) we call $\mathfrak{m}$ a cyclic module and if an arbitary indecomposable A-left or right module is the sum of at most n cyclic modules we call A an algebra of n-cyclic representation type. It is known that A is generalized uniserial if and only if A is of 1-cyclic representation type[1].

In this paper we study the structure of an algebra of 2-cyclic representation type. In order to make the description short we give the next definitions and notations.

(i) If a module or an ideal has only one composition series then we call it *uniserial.*

(ii) If $\frac{Ne_1}{N^2e_1}$ and $\frac{Ne_2}{N^2e_2}$ $(e_1\neq e_2)$ have simple components isomorphic to each other then we call such a component *a vertice component* and $\left\{\frac{N^{j_1}e_1}{N^{j_1+1}e_1},\cdots,\frac{N^{j_r}e_r}{N^{j_r+1}e_r}\right\}$ is called a *chain* if, $\frac{N^{j_\nu}e_\nu}{N^{j_\nu+1}e_\nu}$ and $\frac{N^{j_{\nu+1}}e_{\nu+1}}{N^{j_{\nu+1}+1}e_{\nu+1}}$ $(\nu=1,\cdots,r-1)$ have simple components isomorphic to each other and $\overline{Ae_\nu}$ is not isomorphic to any composition factor of $\frac{Ae_{\nu+1}}{N^{j_{\nu+1}-j_\nu+1}e_{\nu+1}}$ $(j_{\nu+1}\geq j_\nu)$.

(iii) The largest completely reducible part of an A-left (or A-right) module $\mathfrak{m}$ is denoted by $s(\mathfrak{m})$.

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1) See [I] and [II].