CHERN CLASSES OF PROJECTIVE MODULES

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Introduction. In topology, one can define in several ways the Chern class of a vector bundle over a certain topological space (Chern [2], Hirzebruch [7], Milnor [9], Steenrod [15]). In algebraic geometry, Grothendieck has defined the Chern class of a vector bundle over a non-singular variety. Furthermore, in the case of differentiable vector bundles, one knows that the set of differentiable cross-sections to a bundle forms a finitely generated projective module over the ring of differentiable functions on the base manifold. This gives a one to one correspondence between the set of vector bundles and the set of f.g.-projective modules (Milnor [10]). Applying Grauert's theorems (Grauert [5]), one can prove that the same statement holds for holomorphic vector bundles over a Stein manifold.[2]

The purpose of the present paper is to give the Chern class of a f.g.- projective module as an element of the de Rham cohomology of the ring. Thus we establish a completely algebraic treatment of the above cases. Our method of defining the Chern class is the same as that used in differential geometry; thus we obtain a differential geometric approach to the study of projective modules.

In Section 1, we introduce the notion of the trace and its symmetrized form on a finitely generated projective module. For each finitely generated projective module v with constant rank, we construct an exact sequence:

$$0 \to \mathrm{End}\,(v) \to N(v) \to D(R) \to 0$$

where $D(R)$ is the set of derivations of the ring R and $N(v)$ is the set of differential operators. This sequence will play a fundamental role in this paper in achieving a differential-geometric approach to the study of projective

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[2] See also J. P. Serre, "Modules projectifs et espaces fibres a fibre vectorielle" Séminaire P. Dubreil, 1957–58.