NEW FORMULATION OF THE AXIOM OF CHOICE BY MAKING USE OF THE COMPREHENSION OPERATOR

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Introduction

We have introduced in our former work [1] and [2] the comprehension operator "$\{\cdot\}$"[1], which maps every binary relation to a binary relation. The definition of this operator

$$x\{\Gamma\}y \overset{\frown}{=} \forall s(s \in x \equiv s\Gamma y)^{2)}$$

is remarkable in that it can be defined in any formal system having the *membership* relation $\in$, which is hereafter called the ***universal system*** and is denoted by **U**. In this work, we would like to point out that the *axiom of choice, in the strong sense as well as in the weak sense*[3], can be formulated in an extremely simple style by making use of the comprehension operator.

One can make use of our formulations of the axiom of choice in most of the set-theoretical systems, i.e. theories of sets such as the systems of Zermelo [4] and of Fraenkel [5], theories of classes and sets such as the systems of v. Neumann [6], of Bernays [7], and of Gödel [8], and also the theories of objects, **OF** and **OZ** of ourselves, introduced in [2] and [3], respectively.

To unify our notations in this work as far as possible, we would like to establish the following agreement:

We assume a binary relation $\in$ and a field of mathematical objects denoted

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1) The symbol "$\{\cdot\}$" is introduced in [1], and the name "comprehension operator" is introduced in [2]. In this work, we use mostly the same notations as those used in [1]. (Capital Greek letters as meta-symbols for binary relations in general. We use also notations of the forms $\Gamma\varDelta$, $\breve{\Gamma}$, and $\Gamma\wedge\varDelta$ for the relation product of Γ and $\varDelta$, for the inverse relation of Γ, and for the conjunction of Γ and $\varDelta$ (defined as $x(\Gamma\wedge\varDelta)y \overset{\frown}{=} \cdot x\Gamma y \wedge x\varDelta y$), respectively. ($\mathfrak{A} \overset{\frown}{=} \mathfrak{B}$ means here that $\mathfrak{A}$ is defined by $\mathfrak{B}$.)

2) In describing any axiom, any definition, or any theorem, we usually omit the universal quantifiers standing at its top and having the whole formula in its scope.

3) As for the exact meaning of these words, see Sections (2) and (3).