A THEOREM ON FACTORIZABLE GROUPS OF ODD ORDER

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To RICHARD BRAUER on his 60th birthday

Recently, W. Feit [2] obtained some results on factorizable groups of odd order. By using his procedure and applying the theory of R. Brauer [1], we can prove the following theorem similar to that of W. Feit [2]:

THEOREM. *Let G be a factorizable group of odd order such that*

$$G = HM$$

where H is a subgroup of order $3p$, p being a prime greater than 3, and M is a maximal subgroup of G. Then G contains a proper normal subgroup which is contained either in H or in M.

Proof. It is sufficient to prove the theorem in the case in which H is non-abelian. In fact, if H is abelian, then, as $p \neq 3$, the theorem follows immediately from the theorem of W. Feit [2].

Now, assume that no proper normal subgroup of G is contained in M. Suppose that $D = H \cap xMx^{-1} \neq 1$ for some element x in G. If $D = H$, then $H \subseteq xMx^{-1}$. Since every subgroup of G conjugate to M is of the form yMy^{-1} for some element y in H, it follows that H is contained in every subgroup conjugate to M. Hence the intersection of all subgroups conjugate to M is a normal subgroup of G, contained in M. This contradicts our assumption. Thus $D \neq H$. In this case H is represented as the form $H = AD$, where A is a subgroup of prime order which is either p or 3. Since the conjugate subgroup xMx^{-1} is the form yMy^{-1} for an element y in H, $G = A \cdot yMy^{-1}$. By a theorem of T. Ikuta [3], either A is normal in G or yMy^{-1} contains a proper normal subgroup of G. Thus we can assume that $H \cap xMx^{-1} = 1$ for every element x in G.

Let π be the permutation representation of G induced by the subgroup M.

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