NOTE ON GALOIS COHOMOLOGY

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Let K/k be a Galois extension. Formerly the writer studied, in [3], [4], a certain correlation of factor sets in K/k with the norm class group of K/k, and extended it, in [5], to 3-dimensional cocycles. The present note is to study the same relationship for general n-cocycles. As a matter of fact, the constructions which underlie the relationship have become common places in cohomology theory, through the works of French and American authors, and indeed the construction to bring certain (non-Galois) cocycles into the ground field k has been discussed by Baer [1, Theorem C] for general dimensions n under the setting of general group cohomology. Since, however, to assume the triviality of the 2- (and higher) cohomology groups is too strong (and rather meaningless) in the Galois cohomology case,[1] what we may do is, firstly, to assume certain $(n-2)$-, $(n-3)$-, . . . , 2-cocycles appearing successively in the reduction, rather than the corresponding whole cohomology groups, to be ~ 1, as was briefly indicated in [5], or, secondly, to construct, in an also rather well-known way, 2- or 3-cocycles from the original given n-cocycles and apply our former results to them. For an even n these are about all we shall discuss in the following. But, for an odd n there is a certain construction which is similar to the latter of the above constructions but is more general. The writer assumes that this last is somehow worth while to report (Theorem 2); even the above constructions do not seem to the writer to be contained in what has been explicitly stated formerly, in [1] for instance.[2] We shall also extend our former 3-dimensional generalization[3] of the Witt-Akizuki formula to higher dimensions; such possibility being rather expected. The note is a preliminary for a succeeding one in which the writer wants to study the obtained correlations in arithmetical case.

1. *Preliminary observations.* Let K/k be a Galois extension and G be its Galois group. We consider a Galois cocycle $a(\sigma_1, \sigma_2, \dots, \sigma_n)(\sigma_1, \sigma_2, \dots, \sigma_n \in G)$ taking values in the multiplicative group K^* of K;

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1) If the 2-cohomology group in K/k vanishes, so do all (1-, 2- and) 3-, 4-, . . . , n-cohomology groups; see Hochschild-Nakayama [2], § 4.

2) Observe, for example, that the subgroup of $K^*/N^*_{K/k}$ consisting of all elements left invariant under G is in general greater than $k^*/N^*_{k/K}$.

3) See a correction at the end of the present note.