NOTE ON THE HARMONIC MEASURE OF THE ACCESSIBLE BOUNDARY OF A COVERING RIEMANN SURFACE

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Introduction. The following relation was set up in [5] for an open covering Riemann surface $\mathfrak{R}$ with positive boundary over an abstract Riemann surface $\underline{\mathfrak{R}}$:[1]

(1) $\mu(P, \mathfrak{A}(\mathfrak{R})) = \mu(P, \mathfrak{A}(\tilde{\mathfrak{R}})) \geqq \mu(P, \mathfrak{A}(\mathfrak{R}^\infty)) \geqq \mu(P, \mathfrak{A}(\tilde{\mathfrak{R}}^\infty)) \equiv \omega(P)$,

when the universal covering surface $\underline{\mathfrak{R}}'^\infty$ of the projection is not of hyperbolic type; when $\underline{\mathfrak{R}}'^\infty$ is of hyperbolic type this relation is reduced to

(2) $$\mu(P, \mathfrak{A}(\mathfrak{R})) \geqq \mu(P, \mathfrak{A}(\mathfrak{R}^\infty)) \equiv \omega(P).$$

In the present note we shall give some contributions to the clarification of these relations in two special cases.

1. *We suppose first that $\mathfrak{R}$ has a positive boundary, that $\underline{\mathfrak{R}}'^\infty$ is not of hyperbolic type, but that $\mathfrak{R}$ covers a finite number of points $\{\underline{P}_n\}$ of $\underline{\mathfrak{R}}$ only in finite times, where the universal covering surface $(\underline{\mathfrak{R}} - \{\underline{P}_n\})^\infty$ is of hyperbolic type. Under these hypotheses we shall show*

(3) $$\mu(P, \mathfrak{A}(\mathfrak{R}^\infty)) = \mu(P, \mathfrak{A}(\tilde{\mathfrak{R}}^\infty)).$$

For that purpose it is sufficient to prove $\mu(P, \mathfrak{A}(\mathfrak{R}^\infty)) \leqq \mu(P, \mathfrak{A}(\tilde{\mathfrak{R}}^\infty))$ on account of (1).

Map $\mathfrak{R}^\infty$ conformally onto $U: |z| < 1$ and denote by $f(z)$ the function which corresponds to $U \to \mathfrak{R}^\infty \to \mathfrak{R} \to \underline{\mathfrak{R}}$. Let l be an image in U of any determining curve of an accessible boundary point of $\mathfrak{R}$ relative to $\underline{\mathfrak{R}}$. If it is shown that

i) l terminates at a point on $\Gamma: |z| = 1$;[2]

ii) $f(z)$ has an angular limit at every point of $E - E_1$, where E is the image on Γ of $\mathfrak{A}(\mathfrak{R})$ and E_1 is a set of linear measure zero;

iii) E is linearly measurable;

then Lemma in [5] will give $\mu(z, E) \leqq \mu(P, \mathfrak{A}(\tilde{\mathfrak{R}}^\infty))$. On the other hand, the

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[1] We shall follow the definitions and notations in [5] and make use of results in it without proofs.

[2] This point is called an image of a point of $\mathfrak{A}(\mathfrak{R})$.