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ON NON-ANTICIPATIVE LINEAR TRANSFORMATIONS OF GAUSSIAN PROCESSES WITH EQUIVALENT DISTRIBUTIONS

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Let $\xi(t)$, $t \in T$, be a Gaussian process on a set T, and $H = H(\xi)$ be the closed linear manifold generated by all values $\xi(t)$, $t \in T$, with the inner product

$$\langle \eta_1, \eta_2 \rangle = E\eta_1\eta_2 ; \qquad \eta_1, \eta_2 \in H .$$

We suppose that the Hilbert space H is separable.

Let $\mathscr{A}$ be a linear operator on H; we call a random process of the form

$$\eta(t) = \mathscr{A}\xi(t) , \qquad t \in T , \tag{1}$$

a *linear transformation* of the process $\xi(t)$, $t \in T$. One says that a linear transformation $\mathscr{A}$ is *non-anticipative*, if

$$\mathscr{A}H_t(\xi) \subseteq H_t(\xi) , \qquad t \in T , \tag{2}$$

where $H_t(\xi)$ denotes the subspace in H, which is generated by all values $\xi(s)$, $s \leq t$.

Let P be a probability distribution of the Gaussian process $\xi = \xi(t)$, $t \in T$, in some measurable space $(X, \mathfrak{B}, P)$ of «trajectories» $x = x(t)$, $t \in T$, where σ-algebra $\mathfrak{B}$ is generated by all sets $\{x(t) \in B\}$ $(t \in T)$, B are Borel sets on the real line, so P is determined by finite-dimensional distributions of the random process $\xi = \xi(t)$, $t \in T$. Let Q be a probability distribution of the Gaussian process $\eta = \eta(t)$, $t \in T$, represented by the formula (1). According to well known *Feldman's theorem* (see, for example, [1]), *Q is equivalent to P $(Q \sim P)$ if and only if the operator*

$$\boldsymbol{B} = \mathscr{A}^*\mathscr{A} \tag{3}$$

is invertible and $I - \boldsymbol{B} \in S_2$, where S_2 denotes the class of all Hilbert-Schmidt operators in H.

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