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TOPOLOGICAL STABILITY OF SOLENOIDAL AUTOMORPHISMS

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§ 0. Introduction

In [10] A. Morimoto proved that every topologically stable homeomorphism of a compact manifold M has the pseudo-orbit tracing property in the case $\dim(M) \geqslant 2$. Further, in studying relation between the topological stability and other stability of diffeomorphisms, he showed the following

THEOREM A. *Let $\boldsymbol{R}^r$ be the r-dimensional vector group and φ be a group automorphism of $\boldsymbol{R}^r$. Then the following conditions are mutually equivalent;*

(i) *φ is hyperbolic,*
(ii) *φ is expansive,*
(iii) *φ is structually stable,*
(iv) *φ has the pseudo-orbit tracing property,*
(v) *φ is topologically stable.*

The statement further is true for toral automorphisms.

We know (cf. see § 1) that every toral automorphism is contained in the class of solenoidal automorphisms. Thus it will be natural to ask what kind of solenoidal automorphisms have the pseudo-orbit tracing property. Our aim is to investigate this problem by using results in [2] and A. Morimoto [9, 10, 11].

§ 1. A main result and preparatory lemmas

Let $f: X \longleftarrow\!\supset$ be a homeomorphism of a compact metric space (X, d). We denote by $\mathscr{H}(X)$ the group of all homeomorphisms of X. Then $\mathscr{H}(X)$ becomes a complete topological group with the topology given by the metric $d(f, g) = \max\{d(f(x), g(x)), d(f^{-1}(x), g^{-1}(x)) : x \in X\}$ $(f, g \in \mathscr{H}(X))$. We

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