H. Tanimoto
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ON THE BASE FIELD CHANGE OF *P*-RINGS AND *P*-2 RINGS

HIROSHI TANIMOTO

One finds the following example in [3, (34, B)]:

Let k be a field of characteristic p and $\underline{X} = \{X_1, \cdots, X_n\}$ be n-variables over k. Then if $p>0$ and $[k:k^p] = \infty$, $k^p[[\underline{X}]][k]$ is an n-dimensional regular local ring but not a Nagata ring. In particular it is not an excellent ring.

On the other hand, according to [1, Corollary 4.3], $k[[\underline{X}]][l]$ is an excellent ring if l is a separably algebraic field extension of k.

In Section 1 we study when a property such as being excellent ascends by a base field extension.

Conversely in Section 2 we study when such a property as in Section 1 descends by a base field reduction.

§ 1. Notation and definitions

We use the following notation and definitions, following [6]:

Let $\boldsymbol{P}$ be a property meaningful for a noetherian ring and satisfying the following four axioms:

Axioms:
1. If A is regular, then A has $\boldsymbol{P}$.
2. $\boldsymbol{P}$ is a local property.
3. If A is a complete local ring, then $\boldsymbol{P}(A) = \boldsymbol{P}$-locus of A is Zariski open.
4. Let $(A, \mathfrak{m}) \to (B, \mathfrak{n})$ be a faithfully flat local homomorphism. Then $\boldsymbol{P}$ descends from B to A; if both A and $B/\mathfrak{m}B$ have $\boldsymbol{P}$, then $\boldsymbol{P}$ ascends from A to B.

We say that a noetherian ring is a $\boldsymbol{P}$-ring if its formal fibers are geometrically $\boldsymbol{P}$. Then we have:

LEMMA 1. ([6, n. 3, Lemma]). *A noetherian local ring A is a $\boldsymbol{P}$-ring iff, for any finite A-algebra B which is a domain, and for any prime ideal Q of $\hat{B}$ with $Q \cap B = (0)$, the local ring $\hat{B}_Q$ is $\boldsymbol{P}$.*

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