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ON THE FIELDS OF RATIONALITY FOR CURVES AND FOR THEIR JACOBIAN VARIETIES

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Introduction

Throughout the paper, a scheme means a noetherian scheme. By a curve C over a scheme S of genus g, we mean a proper and smooth S-scheme with irreducible curves of genus g as geometric fibres. In the previous paper [15], the author showed that the field of moduli for a non-hyperelliptic curve over a field coincides with the one for its canonically polarized jacobian variety, and in [16], he gave a partial result on the coincidence of the fields of rationality for a hyperelliptic curve and for its canonically polarized jacobian variety. In the present paper, we will discuss the isomorphy of the isomorphism schemes of two curves over a scheme and of their canonically polarized jacobian schemes, by using Oort-Steenbrink's result [12]. Our result is as follows:

Let C and C' be two curves over a scheme S, $P(C)$ and $P(C')$ be their canonically polarized jacobian schemes, respectively. If C and C' are non-hyperelliptic, the canonical map

$$\mathbf{Isom}_S(C, C') \longrightarrow \mathbf{Isom}_S(P(C'), P(C))/\{\pm 1\}$$

is isomorphic. If C and C' are hyperelliptic and S is a scheme over $\mathrm{Spec}(\boldsymbol{Z}[1/2])$, *the canonical map*

$$\mathbf{Isom}_S(C, C') \longrightarrow \mathbf{Isom}_S(P(C'), P(C))$$

is isomorphic.

As a corollary to this result, we get the following:

Let C be a non-hyperelliptic curve over a field of any characteristic or a hyperelliptic curve over a field of characteristic $\neq 2$. If the polarized jacobian variety $P(C)$ is rational over a field k, then C is also rational over k and vice versa. In particular, the field of moduli for C coincides with