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CONTINUOUS VALUATION AND LOGIC

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We consider the continuous valuation of logic, where the certainty of a statement is measured with a number in the closed unit interval $I = [0, 1]$.

The idea originates in continuous logics, which have been investigated from various standpoints, in [1] $\sim$ [4] and [8] $\sim$ [10], for example. More comprehensive information can be seen in [4] and others.

One of the problems which arises in studying continuous logics is the interpretation of the implication; the value of $A \rightarrow B$ is usually defined to be $1 - ([A] \dot{-} [B])$, where $[A]$ represents the value of A. With this interpretation, however, the equality axiom can be dealt with only when the premisses are certain. This situation makes it infeasible to develop set theory along this line.

When working on various valuations, such as Boolean, Heyting and continuous, it has been a traditional practice to consider a logical system which is consistent (and preferably complete) with regards to a given valuation, and to investigate the models of the valid formulas, namely of the formulas which assume constantly the maximal element of the values.

It is interesting, however, to speculate on some theories which abide with the law of classical logic, while allowing deviant valuations. Also, it is natural for us to wish to work on the statements (or events) which are not necessarily valid in the sense state above; namely, we wish to study the situation where the value of a formula is p for any p, reading it as "the degree of certainty of the statement is p", or "the statement is true with certainty p".

More specifically, we work in classical logics, first order and second order, hence in particular $A \rightarrow B$ is interpreted to be $\neg A \vee B$. The value

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