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Nagoya Math. J.
Vol. 77 (1980), 13-23

ON THE MODULE STRUCTURE OF A p-EXTENSION OVER A $\mathfrak{p}$-ADIC NUMBER FIELD

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Throughout this paper, let p be an odd prime. Let k be a $\mathfrak{p}$-adic number field and $\mathfrak{o}$ be the ring of all integers in k. Let K/k be a finite totally ramified Galois p-extension of degree p^n with the Galois group G. Clearly the ring $\mathfrak{O}$ of all integers in K is an $\mathfrak{o}[G]$-module. In the previous paper [4], we studied $\mathfrak{o}[G]$-module structure of $\mathfrak{O}$ in a cyclic totally ramified p-extension, and we have obtained the condition for $\mathfrak{O}$ to be an indecomposable $\mathfrak{o}[G]$-module. In the present paper, we shall prove the following theorem.

THEOREM 1. *Suppose that k contains a primitive p-th root of unity. Let K/k be a totally ramified Galois p-extension of degree p^n such that the extension K/k is not cyclic. Let E be a central idempotent of the group ring $k[G]$ such that $E\mathfrak{O} \subseteq \mathfrak{O}$. Then we have $E = 1$.*

As an immediate consequence of Theorem 1, we have the next theorem.

THEOREM 2. *Let k and K/k be as stated in Theorem 1. In addition, we assume that the extension K/k is abelian. Then the $\mathfrak{o}[G]$-module $\mathfrak{O}$ is indecomposable.*

In § 1, we shall study properties of central idempotent. In § 2, recalling properties of ramification numbers, we shall obtain some inequalities. In § 3, we shall study the special case where the Galois group G is an elementary abelian group of order p^2. In § 4, we shall study the case where the Galois group G is a direct product of two cyclic groups whose orders are p and p^n respectively. In § 5, we shall prove Theorem 1 and Theorem 2.

Received July 25, 1978.