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EVEN CANONICAL SURFACES WITH SMALL K^2, I

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Introduction

Let S be a minimal algebraic surface of general type defined over the complex number field $\mathbf{C}$, and let K denote the canonical bundle. According to [10], we call S a *canonical surface* if the rational map Φ_K associated with $|K|$ induces a birational map of S onto the image X. We denote by $Q(X)$ the intersection of all hyperquadrics through X.

One of the fundamental problems on canonical surfaces is a conjecture of Miles Reid [15, p. 541] which states that a canonical surface satisfies either (1) $K^2 \geq 4p_g - 12$, or (2) the irreducible component of $Q(X)$ containing the canonical image X is of dimension 3. In other words, canonical surfaces with $K^2 < 4p_g - 12$ should have a flavor similar to the exceptions of Enriques-Babbage-Petri's theorem, i.e., trigonal curves and plane quintic curves. Unfortunately, as of now, the conjecture is known to hold only for canonical surfaces with $K^2 = 3p_g - 7$, $3p_g - 6$ (see [4], [8], [1], [10] and [12]).

The present paper is an experiment for Reid's conjecture, considering regular canonical surfaces which are *even*. Here, a compact complex manifold of dimension 2 is called an even surface if the second Stiefel-Whitney class W_2 vanishes [10, §5]. Since they are closed under deformations, even surfaces have their own interest among surfaces of general type; Furthermore, as Horikawa stated in [10, §5], we can rediscover some important lines, e.g., $K^2 = 2p_g - 4$, by considering only even surfaces. This is why we choose them as an experimental material. Let S be an even surface. Since $W_2 = 0$, we can find a line bundle L on S which satisfies $K = 2L$. Such a line bundle L will be referred to as a semi-canonical bundle on S.

In Section 1, even canonical surfaces with $K^2 < 4p_g - 12$, $q = 0$ are classified into three types (I), (II) and (III) according as the nature of the semi-canonical map Φ_L. Namely, we call S a surface of type (I) (resp. type (II)) if Φ_L is a rational map of degree 1 (resp. 3) onto the image, whereas we call it a surface of type (III)

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