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(H, C)-GROUPS WITH POSITIVE LINE BUNDLES

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§ 0. Introduction

Let G be a connected complex Lie group. Then there exists the smallest closed complex subgroup G^0 of G such that G/G^0 is a Stein group (Morimoto [8]). Moreover G^0 is a connected abelian Lie group and every holomorphic function on G^0 is a constant. G^0 is called an (H, C)-group or a toroidal group. Every connected complex abelian Lie group is isomorphic to the direct product $G^0 \times C^m \times C^{*n}$, where G^0 is an (H, C)-group ([7], [9]).

Recently, several interesting results with respect to (H, C)-groups have been obtained (Kazama [5], Kazama and Umeno [6], Vogt [13], [14] and [15]). The set of (H, C)-groups includes the set of complex tori. A complex torus is called an abelian variety if it satisfies Riemann condition. The definition of quasi-abelian variety for (H, C)-groups was given in [2]. In this paper we shall show that the concept of quasi-abelian variety is a natural generalization of abelian variety. Throughout this paper, we assume that $\dim H^1(X, \mathcal{O}) < \infty$ for (H, C)-groups X. Our main result is the following.

Let $X = C^n/\Gamma$ be an (H, C)-group. The following statements are equivalent:

(1) X has a positive line bundle;

(2) X is a quasi-abelian variety;

(3) X is a covering space on an abelian variety;

(4) X is embedded in a complex projective space as a locally closed submanifold.

The above result is well-known for complex tori. For the proof we use the theory of weakly 1-complete manifolds and results of Vogt. We note that implications (2) $\Rightarrow$ (3) and (2) $\Rightarrow$ (4) were obtained by Gherardelli

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