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VECTOR FIELDS ON SOME CLASS OF COMPLETE SYMMETRIC VARIETIES

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§1. Introduction

In the previous papers [6], [7], we show that the set of an algebraic homogeneous space G/P fixed under the action of a maximal torus T can be canonically identified with the coset $W^1 = W/W_1$ of Weyl group W. We find a T invariant Zariski open set near each element $w \in W^1$ and introduce a very nice local coordinate system such that we can express the maximal torus action explicitly. As a result, we become able to apply the study of J. B. Carrell and D. Lieberman [2], [3] to the space G/P and investigate the numerical properties of its characteristic classes and cycles.

The main purpose of this paper is to show that some class of complete symmetric varieties, see the definition in C. DeConcini and C. Procesi [4], [5], have also a nice local coordinate system as above. The canonical compactification of a complex semisimple Lie group of adjoint type belongs to this class. Therefore our results may be related to some combinatorial problems.

§2. Complete symmetric varieties

To fix the terminology which will be needed later, we recall some results about complete symmetric varieties. See [4], [5].

Let G be a simply connected complex semisimple Lie group with an automorphism σ of order two. Let H be the group consisting of all σ fixed elements. We denote by $\mathfrak{g}$ and $\mathfrak{h}$ the Lie algebras which correspond to G and H respectively. Let $\tilde{H}$ be the normalizer of H in G then it is of finite index $[\tilde{H}; H] = 2^s$ for some integer s. For any subgroup H' satisfying $H \subset H' \subset \tilde{H}$, C. DeConcini and C. Procesi call the homogeneous space G/H' a symmetric variety and seek the canonical compactification of $G/\tilde{H}$.

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