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REMARKS ON THE PAPER "TRANSIENT MARKOV CONVOLUTION SEMI-GROUPS AND THE ASSOCIATED NEGATIVE DEFINITE FUNCTIONS"

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Let X be a locally compact and σ-compact abelian group and let $\hat{X}$ denote the dual group of X. We denote by ξ a fixed Haar measure on X and by $\hat{\xi}$ the Haar measure associated with ξ. In [2], we show the following

THEOREM. *Let $(\alpha_t)_{t\geqq 0}$ be a sub-Markov convolution semi-group on X and let ψ be the negative definite function associated with $(\alpha_t)_{t\geqq 0}$. Then $(\alpha_t)_{t\geqq 0}$ is transient if and only if* $\mathrm{Re}\,(1/\psi)$ *is locally $\hat{\xi}$-summable.*

In this theorem, the "if" part is essential. To prove it, we showed the following

PROPOSITION (see Proposition 11 in [2]). *Let n, m be non-negative integers and let $X = R^n \times Z^m$, where R and Z denote the additive group of real numbers and the additive group of integers. Let σ be a probability measure on X with* $\mathrm{supp}\,(\sigma) - \mathrm{supp}\,(\sigma) = X$, *where* $\mathrm{supp}\,(\sigma)$ *denotes the support of σ. Put $\psi(\hat{x}) = 1 - \hat{\sigma}(\hat{x})$ on X, where $\hat{\sigma}$ is the Fourier transform of σ. If* $\mathrm{Re}\,(1/\psi)$ *is locally $\hat{\xi}$-summable, then $\sum_{k=1}^{\infty}(\sigma)^k$ converges vaguely, where $(\sigma)^1 = \sigma$ and $(\sigma)^k = (\sigma)^{k-1} * \sigma$ $(k \geqq 2)$.*

For any positive number p, we put

$$N_p = \frac{1}{p+1}\left(\varepsilon + \sum_{k=1}^{\infty}\left(\frac{1}{p+1}\sigma\right)^k\right),$$

where ε denotes the Dirac measure at the origin. The first main step of the proof in [2] is the following assertion:

$(*)$ There exists $f_0 \in C_K^+(X)$ with $f_0 \neq 0$ such that

$$\left(\left(\frac{1}{2}(N_p + \check{N}_p) - pN_p * \check{N}_p\right) * f_0 * \check{f}_0(0)\right)_{p>0}$$

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