

**THE THEORY OF KM_2O -LANGEVIN EQUATIONS
AND APPLICATIONS TO DATA ANALYSIS (II):
CAUSAL ANALYSIS (1)**

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Dedicated to Professor H. P. McKean, Jr. on his sixtieth birthday

§1. Introduction

It is not too much to say that the problem of finding a cause-and-effect relationship is a fascinating and eternal theme in both natural and social sciences. It is often difficult to decide whether one is the cause of another in two related phenomena, but it is an important problem. It is related to the internal structure of phenomena which generate deterministic or random changes as time passes. We note that the phenomena to be considered are often not deterministic but random. For example, in physical systems such as quantum mechanics or chaotic classical mechanics, it is well known that certain probabilistic reasonings are indispensable.

In several areas in natural science, particularly in physics, causal relations are often discovered by experiments and later explained in theory. However, also in the fields where such methods can not be applied, there sometimes occurs the necessity of deciding a causal direction, or deciding which is the cause and which is the effect in two related phenomena. For example, it has been asserted that Wolfer's sunspot numbers are positively related to the numbers of Lynx and to the economy in Canada ([6], [9], [10], [11], [35], [39]).

Almost all researches in time series analysis have been done on the basis of AR -time series, which has both weak stationarity and finite multiple Markovian property. In particular, such properties have been assumed explicitly or implicitly. However, since we wish to find true intercourse between pure and applied sciences, we should not assume such conditions as those are not yet checked by data analysis, particularly in the problem of finding causal directions. It is desirable

Received December 18, 1990.