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ON p-HYPONORMAL OPERATORS

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Abstract

In this paper, we will give some spectral properties of p-hyponormal operators and two operators T and S on a complex Hilbert space as follows :

(1) T is a p-hyponormal operator which is not quasi-hyponormal.

(2) S is a quasi-hyponormal operator which is not p-hyponormal.

1. Introduction. Let $\mathcal{H}$ be a complex Hilbert space and $B(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. An operator $T \in B(\mathcal{H})$ is said to be hyponormal if $T^*T \geq TT^*$. An operator $T \in B(\mathcal{H})$ is said to be $p-$hyponormal if $(T^*T)^p \geq (TT^*)^p$. Especially, when $p = \frac{1}{2}$, T is called semi-hyponormal. Throughout this paper, let $0 < p \leq \frac{1}{2}$. It is well known that a p-hyponormal operator is q-hyponormal for $q \leq p$ by Lõwner's Theorem. An operator $T \in B(\mathcal{H})$ is said to be quasi-hyponormal if $T^{*2}T^2 \geq (T^*T)^2$. An operator $T \in B(\mathcal{H})$ is said to be paranormal if $\|T^2x\| \geq \|Tx\|^2$ for all unit vectors $x \in \mathcal{H}$. For an operator T, we denote the spectrum and the approximate point spectrum by $\sigma(T)$ and $\sigma_a(T)$, respectively. A point $z \in \mathbf{C}$ in the joint approximate point spectrum $\sigma_{ja}(T)$ if there exists a sequence of unit vectors $\{x_n\}$ in $\mathcal{H}$ such that $(T - z)x_n \to 0$ and $(T - z)^*x_n \to 0$. For an operator $T \in B(\mathcal{H})$, we denote the polar decomposition of T by $T = U|T|$.

We need the following results.

THEOREM A (Th.4 of [6]). *Let T be p-hyponormal. If $Tx = \lambda x$, then $T^*x = \overline{\lambda}x$.*

THEOREM B (Th.8 of [6]). *Let T be p-hyponormal. Then*

$$\sigma_a(T) = \sigma_{ja}(T).$$

Next, let $\mathcal{T}$ be the set of all strictly monotone increasing continuous non-negative functions on $\mathbf{R}^+ = [0, \infty)$. Let $\mathcal{T}_o = \{\varphi \in \mathcal{T} : \varphi(0) = 0\}$. For $\varphi \in \mathcal{T}_o$, the mapping $\tilde{\varphi}$ is defined by

$$\tilde{\varphi}(re^{i\theta}) = e^{i\theta}\varphi(r) \quad \text{and} \quad \tilde{\varphi}(T) = U\varphi(|T|).$$

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