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Structure of certain solvable j-algebras

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Abstract. In this paper we study the stability subgroups of certain solvable Lie groups with respect to the coadjoint action in connection with j-algebras. For this aim we generalize Piatetskii-Shapiro's theory on normal (split solvable) j-algebras. We prove the connectedness of the stabilizers for certain solvable j-algebras. In the last section we give an example of j-simple solvable j-algebras which satisfy our assumtion (1.1) and have rank > 1. Such phenomena do not occur for solvable j-algebras of exponential type which were already treated by I.I.Piatetskii-Shapiro and H.Fujiwara.

1. Introduction and Main Results

We find many literatures which treat the j-algebras in connection with the homogeneous Kähler manifolds (e.g.[2],[3],[8],[11],[14]) or the holomorphically induced unitary representations of Lie groups(e.g. [10],[1],[4],[7],[9],[13]). In this paper we study solvable j-algebras satisfying the condition (1.1), which is given in Theorem 1. Our main motivation to study these j-algebras is to generalize R.Penney's theorem on exponential solvable j-algebras [10],Theorem 2. We believe that our structure theorem is useful to achieve this aim.

Definition. Suppose that $\omega : \mathfrak{g} \to \mathbf{R}$ is a linear functional on a finite dimensional Lie algebra $\mathfrak{g}$ over $\mathbf{R}$. Denote its complexification $\omega^{\mathbf{C}} : \mathfrak{g}^{\mathbf{C}} \to \mathbf{C}$ by the same letter ω. Suppose that $\mathfrak{h}$ is a complex Lie subalgebra of $\mathfrak{g}^{\mathbf{C}}$. The algebra $\mathfrak{h}$ is said to be an *algebraic polarization* of $\mathfrak{g}$ at ω if the following conditions are fulfilled:
i) $\omega([Z_1, Z_2]) = 0$ for every $Z_1, Z_2 \in \mathfrak{h}$. ii) If $Z_0 \in \mathfrak{g}^{\mathbf{C}}$ satisfies $\omega([Z_0, Z]) = 0$ for every $Z \in \mathfrak{h}$, then Z_0 is an element of $\mathfrak{h}$. iii)$\mathfrak{h} + \overline{\mathfrak{h}}$ is a Lie subalgebra of $\mathfrak{g}^{\mathbf{C}}$. An algebraic polarization $\mathfrak{h}$ is said to be *totally complex* if the condition iv) $\mathfrak{h} + \overline{\mathfrak{h}} = \mathfrak{g}^{\mathbf{C}}$ is satisfied. An algebraic polarization $\mathfrak{h}$ at ω is said to be *positive* if v)$\sqrt{-1}\omega([Z, \overline{Z}]) \geq 0$ holds for every $Z \in \mathfrak{h}$. Denote by G the connected, simply connected Lie group with Lie algebra $\mathfrak{g}$. Denote by G_ω the stabilizer of ω,i.e., $G_\omega = \{g \in G : \omega(Ad(g)(X)) = \omega(X) \quad \text{for every} X \in \mathfrak{g}\}$ and by $\mathfrak{g}_\omega$ the Lie algebra of G_ω,i.e.,$\mathfrak{g}_\omega = \{X \in \mathfrak{g} : \omega([X,Y])=0 \text{ for every } Y \in \mathfrak{g}\}$.