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Characterization of generalized surfaces of revolution

Young Ho Kim *

Abstract : We study the so-called generalized surfaces of revolution in a Euclidean space by considering normal sections.

1. Introduction

We define a generalized surface of revolution in (n+1)-dimensional Euclidean space E^{n+1} : Let C be a plane curve in E^{n+1}. A manifold of dimension n generated by revolving C around an axis is said to be a generalized surface of revolution in E^{n+1}. In the present paper, we characterize a generalized surface of revolution in E^{n+1}.

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2. Preliminaries

Let M = (M, x) be an n-dimensional submanifold in m-dimensional Euclidean space E^m, where x is an isometric immersion from M into E^m. Let ∇ and $\tilde{\nabla}$ be the Levi-Civita connections of M and E^m respectively. For any two vector fields X and Y tangent to M, the second fundamental form σ is given by $\sigma(X, Y) = \tilde{\nabla}_X Y - \nabla_X Y$. For a vector field ξ normal to M and X a vector field tangent to M, we may decompose $\tilde{\nabla}_X\xi$ as $\tilde{\nabla}_X\xi = -A_\xi X + \nabla^{\perp}_X\xi$, where $-A_\xi X$ and $\nabla^{\perp}_X\xi$ denote the tangential and normal components of $\tilde{\nabla}_X\xi$, respectively, and $\nabla^{\perp}$ is called the normal connection of the normal bundle $T^{\perp}M$. Let $<, >$ be the scalar product of E^m. Then the Weingarten map A_ξ and the second fundamental form σ have the following relationship : $< A_\xi X, Y > = < \sigma(X, Y), \xi >$ for all vector fields X and Y tangent to M and every normal vector field ξ.

For the second fundamental form σ, we define a covariant derivative $\bar{\nabla}\sigma$ by

$$(2.1) \qquad (\bar{\nabla}_X\sigma)(Y, Z) = \nabla^{\perp}_X\sigma(Y, Z) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z)$$

for vector fields X, Y and Z tangent to M. Let R be the curvature tensor of M. Then the structure equations of Gauss and Codazzi are given by

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