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On the Semigroup Approach to a Class of Space-Dependent Porous Medium Systems

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The main goal of this paper is to treat the nonlinear system of porous medium equations,

$$\text{(PMS)} \qquad \begin{cases} \frac{\partial}{\partial t}u_1(x,t) = \Lambda\phi_1(x,u_1(x,t)) + f_1(x,u_2(x,t),u_1(x,t)) \\ \frac{\partial}{\partial t}u_2(x,t) = \Lambda\phi_2(x,u_2(x,t)) + f_2(x,u_1(x,t),u_2(x,t)) \\ u_1(\cdot,t)|_{\partial\Omega} = u_2(\cdot,t)|_{\partial\Omega} = 0 \ , \ t \geq 0 \\ u_1(x,0) = u_1^0 \ , \quad u_2(x,0) = u_2^0 \quad \text{where } u_1^0, u_2^0 \in L^\infty(\Omega), \end{cases}$$

coupled in the reaction terms $f_1(x,u_2(x,t),u_1(x,t))$ and $f_2(x,u_1(x,t),u_2(x,t))$. Here Ω is a bounded open domain in $\mathbb{R}^n$, and its boundary $\partial\Omega$ is sufficiently smooth. We use the symbol Λ to represent a strongly elliptic operator such as the Laplace operator $\triangle$ on a given domain in $L^1(\Omega)$, defined in 2.9. In order to handle this system, we proceed in a number of stages.

Firstly, we consider the following single equation

$$\text{(PME)} \qquad \begin{cases} \frac{\partial}{\partial t}u(x,t) = \Lambda\phi(x,u(x,t)) + f(x,u(x,t)) \\ u(\cdot,t)|_{\partial\Omega} = 0 \\ u(x,0) = u_0(x) \ , \quad u_0 \in L^1(\Omega). \end{cases}$$

The functions ϕ and f are assumed to be continuous on $\overline{\Omega}\times\mathbb{R}$, and to satisfy certain natural assumptions which will be detailed in 2.6. Under those conditions we shall show that the theory of nonlinear semigroups can be applied to prove the existence of unique solutions in a generalized sense to (PME). This is done by first considering the semilinear equation $-\Lambda v + Gv = w$ for $w \in L^1$, G being the composition operator defined by $g(\cdot,v) = f(\cdot,\phi^{-1}(v))$, under appropriate conditions on g. Existence of solutions v to this simpler equation allows the *range condition* (RC) from nonlinear semigroup theory to be proven. Hence, by showing that the nonlinear operator $A = \Lambda + F$ is dissipative, the result is obtained.