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Minimal singular compactifications of the affine plane

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Abstract. Let X be a minimal compactification of the complex affine plane $\mathbf{C}^2$. In this paper, we show that X is a log del Pezzo surface of rank one and determine the singularity type of X in the case where X has at most quotient singularities.

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0 Introduction

A normal compact complex surface X is called a *compactification* of the complex affine plane $\mathbf{C}^2$ if there exists a closed subvariety Γ of X such that $X - \Gamma$ is biholomorphic to $\mathbf{C}^2$. We denote simply the compactification by the pair (X, Γ). A compactification (X, Γ) of $\mathbf{C}^2$ is said to be *minimal* if Γ is irreducible.

Remmert–Van de Ven [26] proved that if (X, Γ) is a minimal compactification of $\mathbf{C}^2$ and X is smooth then $(X, \Gamma) = (\mathbf{P}^2, \text{line})$. Brenton [3], Brenton–Drucker–Prins [4] and Miyanishi–Zhang [21] studied minimal compactifications of $\mathbf{C}^2$ with at most rational double points and proved the following results.

Theorem 0.1 (cf. [3], [4] and [21]) *If (X, Γ) is a minimal compactification of $\mathbf{C}^2$ and X has at most rational double points, then X is a log del Pezzo surface of rank one (for the definition, see Definition 2.1). Further, if* Sing $X \neq \emptyset$, *then the singularity type of X is given as one of the following:*

$$A_1,\ A_1 + A_2,\ A_4,\ D_5,\ E_6,\ E_7,\ E_8.$$