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L^2 THEORY FOR THE OPERATOR $\Delta + (k \times x) \cdot \nabla$ IN EXTERIOR DOMAINS

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ABSTRACT. In exterior domains of $\mathbb{R}^3$, we consider the differential operator $\Delta+(k\times x)\cdot\nabla$ with Dirichlet boundary condition, where k stands for the angular velocity of a rotating obstacle. We show, among others, a certain smoothing property together with estimates near $t=0$ of the generated semigroup (it is not an analytic one) in the space L^2. The result is not trivial because the coefficient $k \times x$ is unbounded at infinity. The proof is mainly based on a cut-off technique. The equation $\partial_t u = \Delta u + (k \times x) \cdot \nabla u$ can be taken as a model problem for a linearized form of the Navier-Stokes equations in a domain exterior to a rotating obstacle. This paper is a step toward an analysis of the Navier-Stokes flow in such a domain.
Key words and phrases: differential operators with unbounded coefficients, exterior domains, semigroups, smoothing effects.

1. Introduction and statement of main results

Let $\mathcal{O} \subset \mathbb{R}^3$ be a compact obstacle which is bounded by a smooth surface Γ. In the exterior domain $\Omega = \mathbb{R}^3 \setminus \mathcal{O}$ we consider the initial boundary value problem

$$(1.1) \qquad \begin{cases} \partial_t u = \Delta u + (k \times x) \cdot \nabla u, & x \in \Omega,\ t > 0, \\ u(x,t) = 0, & x \in \Gamma,\ t > 0, \\ u(x,t) \to 0, & |x| \to \infty,\ t > 0, \\ u(x,0) = a(x), & x \in \Omega, \end{cases}$$

where $k = (0,0,1)^T$, so that $k \times x = (-x_2, x_1, 0)^T$. The aim of the present paper is to establish some fundamental properties for the differential operator $\Delta+(k\times x)\cdot\nabla$ in exterior domains. It is proved that the operator with homogeneous Dirichlet boundary condition generates a semigroup having a certain smoothing property and enjoys an elliptic regularity estimate in the space L^2.

Let us explain the motivation of this study. Assume that the exterior domain Ω is occupied by a viscous incompressible fluid and that the obstacle $\mathcal{O}$ is rotating about the x_3-axis with angular velocity k. We then consider the fluid motion governed by the Navier-Stokes equation in the domain $\Omega(t) = \{O(t)x;\ x \in \Omega\}$, where

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