SO(3)-action and 2-torus action on homotopy complex projective 3-spaces

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(Received May 20, 1975)

Introduction

In this note, we shall consider SO(3)-action and 2-torus action on homotopy complex projective 3-space with the view of proving that any exotic homotopy complex projective 3-space admits no effective SO(3)-action nor 2-torus action. In this direction the following results are known.

1. *A homotopy complex projective* 3*-space (abbreviated by* hCP_3*) which admits an effective* 3*-torus action is diffeomorphic to the standard complex projective* 3*-space* CP_3. ([8])

2. *An* hCP_3 *which admits an effective* 1*-torus action with fixed point set of* 2 *components is diffeomorphic to* CP_3. ([11])

3. *An* hCP_3 *which admits n-dimensional compact connected Lie group action* ($n \geqq 6$) *is diffeomorphic to* CP_3. ([4])

In this note we shall prove the following

THEOREM. *If an* hCP_3 *admits an effective* $SO(3)$*-action or* 2*-torus action, then it is diffeomorphic to* CP_3.

In the following all actions are assumed to be differentiable.

1. Statement of results

First we shall consider 2-torus action. Let G be a 2-torus and t_1, t_2 denote the standard complex 1-dimensional representations of G. Then it is well known that the complex representation ring $R(G)=Z[t_1, t_2, t_1^{-1}, t_2^{-1}]$.

LEMMA 1. *Let* ϕ_1 *and* ϕ_2 *be complex* 1*-dimensional representations of* G. *Put* $\phi_1=t_1^a t_2^b$ *and* $\phi_2=t_1^c t_2^d$, *where* a, b, c, d, *are integers. Then if* $\ker \phi_1 \cap \ker \phi_2=1$, *we have* $ad-bc=1$.

PROOF. Assume the contrary. Then $ad-bc=e\neq\pm 1$ and e is not zero. It is easy to see that there are integers k, l such that at least one of $(kd-lb)/e$, $(la-kc)/e$ is not integer. Hence there exist real numbers θ and λ such that $a\theta+b\lambda\equiv 0\ (2\pi)$, $c\theta+\lambda d\equiv 0\ (2\pi)$