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004

# NAVIER-STOKES APPROXIMATIONS TO 2D VORTEX SHEETS IN HALF PLANE*

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**Abstract.** This paper concerns the two-dimensional Euler equations with vortex-sheets initial data in half plane and in the domain $\Omega = \{(x_1, x_2) : x_2 \geq \gamma(x_1)\}$, $\gamma(x_1) = 0$ for $|x_1| \geq x_0$, $x_0$ is a fixed constant, and $\gamma(x_1)$ is a sufficient smooth and simple curve. The Navier-Stokes approximations are constructed in this paper and by means of vanishing the viscosity, the global existence of weak solutions is obtained under the assumption that the initial vorticity is of one-sign. Navier boundary conditions are applied when constructing the Navier-Stokes approximations.

**Key words.** Vanishing viscosity limit, Euler equations, vortex-sheets data, Navier-Stokes approximations.

**AMS subject classifications.** 76B47, 35Q30

**1. Introduction.** We consider the following two-dimensional incompressible Euler equations

$$\begin{cases} \partial_t u + u \cdot \nabla u + \nabla p = 0 & x \in \mathbb{H},\ t > 0, \\ \mathrm{div} u = 0 & x \in \mathbb{H},\ t > 0, \\ |u(x,t)| \to 0 & |x| \to \infty, \end{cases} \tag{1.1}$$

where $\mathbb{H} = \{(x_1, x_2) : x_2 \geq 0\}$. Define $\Gamma \doteq \partial\mathbb{H} = \{x_2 = 0\}$. The unknown functions $p = p(x,t)$ and $u = (u_1(x,t), u_2(x,t))$ represent the pressure and velocity fields function, respectively.

The initial and boundary conditions of (1.1) are imposed as

$$u(x, t = 0) = u_0 \ \ x \in \mathbb{H}, \tag{1.2}$$

and

$$u \cdot n = 0 \ \text{ on } \ \Gamma. \tag{1.3}$$

In (1.3), $n$ means the unit normal vector of $\Gamma$.

The vorticity of the velocity $u(x,t)$ is denoted by $\omega(x,t) = \mathrm{curl} u$ and the initial vorticity is given by $\omega_0(x) = \mathrm{curl} u_0$. Roughly speaking, for a general domain $\Omega \subseteq \mathbb{R}^2$, the initial data of (1.1) are called vortex-sheets data if the initial velocity is locally square integrable and the initial vorticity is a finite Radon measure,that is, $u_0 \in$

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