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SMOOTHING PROPERTIES OF SOME WEAK SOLUTIONS OF THE BENJAMIN–ONO EQUATION

MICHAEL MUDI TOM
Department of Mathematics, Pennsylvania State University, University Park, PA 16802 USA

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Abstract. Weak solutions corresponding to initial data in $H^{n/2}(\mathbb{R})$, for $n = 2$ or 3, of the Benjamin-Ono equation are shown to exhibit a local smoothing effect in the spatial variable of $\frac{1}{2}$ a derivative, almost everywhere in time. Similar results are also shown to hold true for the intermediate long-wave equation and Smith's equation.

1. Introduction. In this note we consider the initial value problem for the Benjamin-Ono equation

$$u_t + uu_x - Hu_{xx} = 0, \quad u(x,0) = u_0(x), \quad x \in \mathbb{R}, \ t > 0 \tag{1.1}$$

where H connotes the Hilbert transform

$$(Hf)(x) = \text{p.v.}\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(y)}{x-y}\,dy = F^{-1}(-i\,\text{sgn}(\zeta))\hat{f}(\zeta))$$

where $\hat{}$ and F^{-1} denote the Fourier transform and its inverse, respectively.

This equation arises in the study of internal waves in deep, stratified fluids (see [2], and [12]). Recently, Ponce [13] showed that solutions of (1.1) corresponding to initial data in H^s where $s > \frac{3}{2}$ have $s + \frac{1}{2}$ derivatives locally in L^2, almost everywhere in time. He was also able to demonstrate that weak solutions corresponding to initial data in H^1 have their $\frac{5^{\text{th}}}{4}$-derivative lying locally in L^2 for almost all t. By combining Ponce's techniques with some recent methods which Bona and Saut [3] applied to the Korteweg-de Vries equation, a sharper theorem of local smoothing may be derived for both H^1 and $H^{3/2}$ weak solutions of (1.1). Using this result and a perturbation argument then leads to similar results for the intermediate depth equation for internal waves and Smith's equation for continental shelf waves (see [1]).

This paper is organized as follows. Notation and inequalities that will be used are contained in Section 2. In Section 3, we prove the appropriate local smoothing result for solutions of the IVP (1.1) with data $u_0 \in H^{n/2}$ for $n = 2$ or 3. The local smoothing results for the intermediate long wave equation and Smith's equation are provided in Section 4.

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