Differential and Integral Equations, Volume 6, Number 1, January 1993, pp. 83–91.

AN OSCILLATION CRITERION FOR SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH ITERATED INTEGRAL AVERAGES

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Abstract. Consider the second order nonlinear differential equation $y'' + a(t)f(y) = 0$, where $a(t) \in C[0,\infty)$, $f(y) \in C^1(-\infty,\infty)$, $f'(y) \geq 0$ and $yf(y) > 0$ for $y \neq 0$. Furthermore, $f(y)$ also satisfies either a superlinear or a sublinear condition, which covers the prototype nonlinear function $f(y) = |y|^\gamma \mathrm{sgn} y$, with $\gamma > 1$ or $0 < \gamma < 1$,respectively, known as the Emden-Fowler case. The coefficient $a(t)$ is allowed to be negative for arbitrarily large values of t. An earlier result in the linear case due to Kamenev involving iterated integral averages of $a(t)$ is extended to the general nonlinear equation subject to certain nonlinear conditions on $f(y)$. In particular, the result applies to the Emden-Fowler case for all $\gamma > 0$.

1. Consider the second order nonlinear differential equation

$$y'' + a(t)f(y) = 0, \quad t \in [0,\infty), \tag{1}$$

where $a(t) \in C[0,\infty)$ and $f(y) \in C^1(-\infty,\infty)$, $f'(y) \geq 0$ for all y, and $yf(y) > 0$ if $y \neq 0$. The prototype of equation (1) is the so-called Emden-Fowler equation

$$y'' + a(t)|y|^\gamma \mathrm{sgn} y = 0, \quad \gamma > 0. \tag{2}$$

Here we are interested in the oscillation of solutions of (1) when $f(y)$ satisfies, in addition, the sublinear condition

$$0 < \int_0^\varepsilon \frac{dy}{f(y)}, \int_{-\varepsilon}^0 \frac{dy}{f(y)} < \infty \quad \text{for all} \quad \varepsilon > 0, \tag{F$_1$}$$

which corresponds to the special case $f(y) = |y|^\gamma \mathrm{sgn} y$ when $0 < \gamma < 1$, and also the superlinear condition

$$0 < \int_\varepsilon^\infty \frac{dy}{f(y)}, \int_{-\infty}^{-\varepsilon} \frac{dy}{f(y)} < \infty \quad \text{for all} \quad \varepsilon > 0, \tag{F$_2$}$$

which corresponds to the special case $f(y) = |y|^\gamma \mathrm{sgn} y$ when $\gamma > 1$. The coefficient $a(t)$ is allowed to be negative for arbitrarily large values of t. Under these circumstances, in general not every solution to the second order nonlinear differential

Received for publication November 1991.
AMS Subject Classification: 34C10, 34C15.