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AN INVERSE PROBLEM FOR AN ABSTRACT NONLINEAR PARABOLIC INTEGRODIFFERENTIAL EQUATION

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Abstract. We consider a nonlinear abstract parabolic integrodifferential equation having a pair of time-dependent kernels which are unknown. We want to recover such kernels by using overdetermined data. We prove a local (in time) existence and uniqueness result. This result is obtained by using the analytic semigroup theory to reduce the inverse problem to a system of nonlinear Volterra equations of the second kind. Then the equivalent problem is solved via the Contraction Principle. Moreover, an application to the nonlinear heat conduction with memory is shown.

1. Introduction. Let us consider the nonlinear abstract Cauchy problem

$$u'(t)+(h*u')(t)+A(u(t))+(k*B(u))(t)=f(t), \quad \text{for a.e. } t\in(0,T),\ T>0, \tag{1.1}$$

$$u(0)=u_0, \tag{1.2}$$

where A, B are nonlinear operators, h, k, f are given functions defined on $(0,T)$, and u_0 is a given element. Here and in the sequel $*$ denotes the convolution product with respect to time.

Equations like (1.1) describe, for instance, the heat conduction in materials with memory (see, e.g., [11, 16]). Due to these applications, the Cauchy problem (1.1–2) has been widely investigated by many authors both from the theoretical and the numerical viewpoint (see, e.g., [1, 2, 3, 7, 9, 11, 12, 15, 16, 18] and the references therein).

Since the *relaxation* or *memory functions* h and k are *a priori* unknown or scarcely known in the applications (see, e.g., [6, 16]), the problem (1.1–2) is essentially underdetermined and the question that arises is

how does one recover h and k in addition to u ?.

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