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EXISTENCE OF SOLUTIONS FOR NONLINEAR BOUNDARY VALUE PROBLEMS AND INJECTIVITY OF RELATED LINEAR DIFFERENTIAL OPERATORS

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1. Introduction. Consider the nonlinear boundary value problem

$$x' = F(t,x), \tag{1}$$
$$\Gamma(x) = r, \tag{2}$$

where $F : I \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies Caratheodory conditions on the compact interval I; i.e., F is measurable in t for all x in $\mathbb{R}^n$ and continuous in x for almost every $t \in I$. We assume that Γ is a linear continuous mapping from the space $C(I, \mathbb{R}^n)$ of n-dimensional vector functions defined in I into $\mathbb{R}^n$. We are interested in the case where F has linear growth, which amounts to considering problems of the form

$$x' = G(t,x)x + R(t,x), \tag{3}$$
$$\Gamma(x) = r, \tag{4}$$

where $G(t,x)$ is a $n \times n$ matrix and R is a sublinear term, by which we mean that $R(t,x) = o(|x|)$ for $|x|$ going to infinity, uniformly in t. This situation is fairly easy to deal with when the linear homogeneous problems associated to (3), (4), namely the problems

$$u' = G(t, x(t))u,$$
$$\Gamma(u) = 0,$$

have only the trivial solution for any $x \in C(I, \mathbb{R}^n)$. The study of such situations goes back at least to Z. Opial [6] and Theorem 1 below can be deduced from his results in [6]. In Theorem 1, problem (3), (4) will be related to a family of linear problems

$$x' = S(t)x, \tag{5}$$
$$\Gamma(x) = 0, \tag{6}$$

the matrices $S(\cdot)$ belonging to some interval $[M - A, M + A]$. By this, we mean that the elements $s_{ij}(\cdot)$, $\mu_{ij}(\cdot)$, $\alpha_{ij}(\cdot)$ of S, M, A, respectively, are L^1-functions from I into $\mathbb{R}$, satisfying the conditions

$$|s_{ij}(t) - \mu_{ij}(t)| \leq \alpha_{ij}(t) \quad \text{for a.e. } t \in I, \quad \text{for } i,j = 1, \dots, n.$$

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