Differential and Integral Equations, Volume 7, Number 4, July 1994, pp. 1063 – 1082.

GROUND STATES OF A QUASILINEAR EQUATION

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To the memory of Peter Hess

1. Introduction. In 1989, Chipot and Weissler [2] proposed the equation

$$u_t = \Delta u - |\nabla u|^q + \lambda |u|^{p-1} u, \quad \lambda > 0 \tag{1.1}$$

for the study of blow-up phenomena for parabolic equations. The term $|u|^{p-1}u$ in this equation has the tendency to cause blow-up, whilst the term $-|\nabla u|^q$ just opposes this tendency. It was the competition between these terms, for different values of the parameters p and q, which they studied. Whereas [2] was mainly focused on the study of (1.1) in bounded domains in $\mathbb{R}^N$, Alfonso and Weissler [1] and Serrin and Zou [4] studied (1.1) in unbounded domains. In particular, in [4] a very extensive analysis was given of nonnegative radially symmetric equilibrium solutions of (1.1) in $\mathbb{R}^N$ – often referred to as *ground-states* when $u(x) \to 0$ as $x \to \infty$ – in different regions of the (p, q) parameter space and for $N > 1$.

In this paper we study the existence, uniqueness and asymptotic behavior of ground states when $N = 1$. Thus, we consider the problem

$$\text{(I)} \qquad \begin{cases} u'' - |u'|^q + \lambda u^p = 0 & \text{on} \quad \mathbb{R} & \text{(1.2a)} \\ u \geq 0, \ u \not\equiv 0 & \text{on} \quad \mathbb{R} & \text{(1.2b)} \\ u(x) \to 0 & \text{as} \quad x \to \pm\infty. & \text{(1.2c)} \end{cases}$$

About the parameters p and q we assume initially merely that they are positive.

It was shown in [2] that if

$$p > 1, \quad q > \frac{2p}{p+1}, \tag{1.3}$$

Received September 1993.