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CERTAIN SYSTEMS WITH PIECEWISE CONSTANT FEEDBACK CONTROLS WITH A TIME DELAY

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The rate of growth of some quantities involved in certain biological or chemical processes can often be assumed to be positive and proportional to their size when this size is small, but for large values of the size, this rate becomes negative. For example, the logistic equation

$$dN/dt = N(b - N),$$

where b is a positive constant, proposed as a model for the population size $N(t)$ of a biological species growing in a constant environment with limited resources for growth, clearly exhibits such behavior. In particular, it is easy to see that all positive solutions $N(t) \to b$ as $t \to \infty$. This system can be regarded as a feedback control system with the second N on the right the feedback control term.

For some population processes, however, this simple logistic model may not be valid; for example, if the population size exhibits oscillatory behavior, this model fails, since all solutions $N(t)$ of this equation must be monotonic. If the control term is entered into the equation with a time delay, i.e., if the model is given by the delay differential equation

$$dN/dt = N(t)(b - N(t-r)),$$

where $r > 0$ then such oscillatory solutions are possible. It is also known that for such equations, not all positive solutions may approach b as $t \to \infty$, and in fact, some of these solutions may exhibit chaotic behavior. This has been found for solutions of

$$dN/dt = N(t)(b - N([t])), \quad t > 0,$$

where $[t]$ denotes the greatest integer in t, for b sufficiently large; cf. [1] and Theorem 1.1 in the present paper. We note that in this equation, the feedback control term $N([t])$ is piecewise constant.

In this paper we consider equations of the form

$$dx/dt = x(t)\Big(b - \sum_{j=0}^{n} a_j x([t-j])\Big), \quad t > 0, \tag{FD}$$

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