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ON THE EXISTENCE OF EXTREMAL WEAK SOLUTIONS FOR A CLASS OF QUASILINEAR PARABOLIC PROBLEMS

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Abstract. The existence of the greatest and least solution (in the sense of an underlying partial ordering) enclosed by assumed upper and lower solutions is proved by showing that the solution set between the given upper and lower solution is directed and by applying Zorn's lemma. Our approach allows treatment of rather general quasilinear parabolic initial boundary value problems under low regularity assumptions on the data. In this way our result extends numerous earlier results on extremal solutions obtained by using monotone iteration technique on the one hand and Akô's method on the other hand.

1. Introduction. Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with Lipschitz boundary $\partial\Omega$, $Q = \Omega \times (0,\tau)$ and $\Gamma = \partial\Omega \times (0,\tau)$, $\tau > 0$. In this paper we consider weak solutions of the following initial boundary value problem (IBVP)

$$\left.\begin{aligned} \frac{\partial u(x,t)}{\partial t} + Au(x,t) + Fu(x,t) &= h(x,t) \quad \text{in } Q, \\ u(x,t) &= g(x,t) \quad \text{on } \Gamma, \\ u(x,0) &= \psi(x) \qquad \text{in } \Omega, \end{aligned}\right\} \tag{1.1}$$

where A is a second-order quasilinear elliptic differential operator of divergence form, i.e.,

$$Au(x,t) = -\sum_{i=1}^{N} \frac{\partial}{\partial x_i} a_i(x,t,\nabla u(x,t))$$

and F is the Nemytskij operator associated with a function $f\colon Q \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ given by $Fu(x,t) = f(x,t,u(x,t),\nabla u(x,t))$ with $\nabla u = (\frac{\partial u}{\partial x_1},\dots,\frac{\partial u}{\partial x_N})$.

The existence of classical extremal solutions has been proved previously by Sattinger in [18] using a monotone iteration scheme which was possible because of a linear differential operator A and a gradient independent function f satisfying a one sided Lipschitz continuity with respect to u. Bebernes and Schmitt [2] extended Sattinger's result in that they admit a much larger class of nonlinearities f which may depend upon the gradient ∇u (though a Nagumo type growth condition with respect to this variable is required) and f need not be Lipschitz continuous with respect to u. Using a different approach (an approach patterned after methods employed by Akô [1] in the study of elliptic boundary value problems) these authors

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