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STABILITY PROPERTY FOR AN INTEGRODIFFERENTIAL EQUATION

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Abstract. In order to obtain the existence of an almost periodic solution for an almost periodic integrodifferential equation, we consider a certain stability property, which is referred to as the stability under disturbances from hull, and show that for a periodic integrodifferential equation, this stability property is equivalent to uniform stability.

For ordinary differential equations and functional differential equations with infinite delay, it is known that the existence of an almost periodic solution of an almost periodic system can be obtained under the total stability condition. For references, see [1], [4], [6] and [9].

In order to obtain an existence theorem for almost periodic solutions in ordinary differential equations, Sell [7] introduced a new stability concept which is referred to as the stability under disturbances from hull. This stability property is weaker than total stability (cf. [5, 9]). For periodic systems, Yoshizawa [8] has shown that uniform stability and stability under disturbances from hull are equivalent.

In this article, we shall discuss the relationship between stability under disturbances from hull and total stability in an almost periodic integrodifferential equation, and show that for a periodic integrodifferential equation, uniform stability and stability under disturbances from hull are equivalent.

We shall consider an almost periodic system of integrodifferential equations

$$\dot{x}(t) = f(t, x(t)) + \int_{-\infty}^{0} F(t, s, x(t+s), x(t))\, ds, \tag{1}$$

where $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous and is almost periodic in t uniformly for $x \in \mathbb{R}^n$, and $F(t, s, x, y)$ is continuous on $\mathbb{R} \times (-\infty, 0] \times \mathbb{R}^n \times \mathbb{R}^n$ and is almost periodic in t uniformly for $(s, x, y) \in \mathbb{R}^* = (-\infty, 0] \times \mathbb{R}^n \times \mathbb{R}^n$. For the definition and the properties of almost periodic functions with parameters, see [9]. If x is a function defined on $(-\infty, a)$, x_t is defined by the relation $x_t(s) = x(t+s)$, $-\infty < s \leq 0$. Let $|x|$ be any norm of x in $\mathbb{R}^n$. BC denotes the vector space of bounded continuous functions mapping $(-\infty, 0]$ into $\mathbb{R}^n$, and for any ϕ, $\psi \in BC$, we set

$$\rho(\phi, \psi) = \sum_{j=1}^{\infty} \rho_j(\phi, \psi)/2^j[1 + \rho_j(\phi, \psi)],$$

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