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NONEXISTENCE OF PERIODIC SOLUTIONS OF A COMPLEX RICCATI EQUATION[1]

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In this note we construct an example of a complex-valued differential equation of the kind

$$z' = z^2 + p(t), \tag{1}$$

with $p : \mathbb{R} \to \mathbb{C}$ smooth and 2π-periodic, and such that (1) has not 2π-periodic solutions.

The problem of existence or nonexistence of periodic solutions for equation (1) has been recently posed by J. Mawhin in [3] and it is this question that motivates our paper. Other related references are [1], [2]. The construction of $p(t)$ will follow after two lemmas. They will be proved later.

Lemma 1. *Let $z(t)$ be the solution of the initial value problem*

$$z' = z^2 + 1, \quad z(0) = z_0, \quad (z_0 \in \mathbb{C}). \tag{2}$$

Then,

i) *If $z_0 \in \mathbb{R}$, $z(t)$ is not defined in $[0, \pi]$.*
ii) *If $z_0 \in \mathbb{C} - \mathbb{R}$, $z(t)$ is π-periodic.*

Lemma 2. *There exists a function $s : \mathbb{R} \to \mathbb{R}$ that is of class C^∞, π-periodic and satisfies*

$$s(0) = 1, \quad s^{(n)}(0) = 0, \quad n = 1, 2, \dots$$

and such that the equation

$$z' = z^2 + s(t) \tag{3}$$

has only real π-periodic solutions; that is, every π-periodic solution of (3) *satisfies $z(t) \in \mathbb{R}$, $\forall t \in \mathbb{R}$.*

Remark. If $z(t)$ is a solution of (3) with $z(t_0) \in \mathbb{R}$, for some t_0, then $z(t)$ is a real solution.

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