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ENTROPY CONDITION FOR A QUASILINEAR HYPERBOLIC EQUATION WITH HYSTERESIS

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Abstract. A quasilinear hyperbolic equation with hysteresis is studied. For the integral solution of this equation we derive an entropy condition of the type introduced by Kružkov.

1. Introduction

In this paper we study a hyperbolic equation of first order of the form

$$u_t + [\phi(u)]_x = 0, \qquad u(0) = u_0 \tag{1.1}$$

and the corresponding quasilinear hyperbolic equation with hysteresis

$$\frac{\partial}{\partial t}(u+w) + \sum_{j=1}^{N} \frac{\partial}{\partial x_j}(b_j u) + cu = f, \tag{1.2}$$

where $w = \mathcal{F}(u)$ represents hysteresis.

It is well known that even for ϕ and u_0 smooth (1.1) exhibits singularities in a finite time. To be able to continue the solution, one has to pass to a generalized concept of weak solutions where discontinuities are allowed. Weak solutions are in general not uniquely determined by the data, and further physically motivated conditions have to be prescribed. The simplest one is an entropy condition stating that the entropy of the system must be decreasing, generalized by Olejnik [5]. A different condition was derived by Kružkov [4] and there are many others. Inspired by Kružkov's work, Crandall [1] shows that the unique integral solution of (1.1), constructed by the method of nonlinear semigroups, satisfies an entropy condition derived by Kružkov. In the first section we give a brief overview of their results.

We consider Equation (1.2) coupled with a generalized play or Prandtl-Ishlinskii operator of play type. It was expected (see [9]) that the integral

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