Commun. Math Phys 166, 221–235 (1994)

Communications in Mathematical Physics

On the Newtonian Limit of General Relativity

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Received: 25 March 1992/in revised form April 14 1994

Abstract: We establish rigorous results about the Newtonian limit of general relativity by applying to it the theory of different time scales for non-linear partial differential equations as developed in [4,1,8]. Roughly speaking, we obtain a priori estimates for solutions to the Einstein's equations, an intermediate, but fundamental, step to show that given a Newtonian solution there exist continuous one-parameter families of solutions to the full Einstein's equations – the parameter being the inverse of the speed of light – which for a finite amount of time are close to the Newtonian solution. These one-parameter families are chosen via an *initialization procedure* applied to the initial data for the general relativistic solutions. This procedure allows one to choose the initial data in such a way as to obtain a relativistic solution close to the Newtonian solution in any a priori given Sobolev norm. In some intuitive sense these relativistic solutions, by being close to the Newtonian one, have little extra radiation content (although, actually, this should be so only in the case of the characteristic initial data formulation along future directed light cones).

Our results are local, in the sense that they do not include the treatment of asymptotic regions; global results are admittedly very important – in particular they would say how differentiable the solutions are with respect to the parameter – but their treatment would involve the handling of tools even more technical than the ones used here. On the other hand, this local theory is all that is needed for most problems of practical numerical computation.

1. Introduction

As it has been suggested through the extended history of the treatment of the newtonian limit of relativity, slow-motion corrections should be naturally obtained by suitable approximation schemes starting from the newtonian gravitation theory.

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