Commun. Math. Phys. 161, 283–287 (1994)

Communications in Mathematical Physics


# A Lower Estimate for the Modified Steiner Functional

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Received: 15 April 1993

**Abstract:** We prove inequality (1) for the modified Steiner functional $A(M)$, which extends the notion of the integral of mean curvature for convex surfaces. For the proof, we also establish an expression for $A(M)$ in terms of an integral over all hyperplanes intersecting the polyhedral surface $M$.

## 1. Introduction

In the articles [1, 4, 5] the authors suggest a new version of string theory, which can be considered as a natural extension of the Feynman–Kac integral over paths to an integral over surfaces. Both amplitudes coincide in the case when the surface degenerates into a single particle world line.

The string has been conjectured to describe a wide variety of physical phenomena, including strong interaction, the three dimensional Ising model, and unified models incorporating gravity. The Feynman integral for the string is just the partition function for the randomly fluctuating surfaces, and in this statistical approach the surface is associated with a connected polyhedral surface embedded in euclidean space.

To prove the convergence of the partition function for this new string, the authors of [4, 5] require a lower estimate for the action $A(M)$ on which the theory is based. The purpose of the present note is to prove the inequality

$$A(M) > 2\pi\Delta , \tag{1}$$

where $A(M)$ is the modified Steiner functional as introduced in [1, 4, 5] and $\Delta$ is the diameter of the polyhedral surface $M$ in $\mathrm{R}^d$. We also establish an expression for $A(M)$ in terms of an integral over all hyperplanes intersecting the polyhedral surface $M$.