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On the BiHamiltonian Structure of the Supersymmetric KdV Hierarchies. A Lie Superalgebraic Approach

Carlo Morosi[1], **Livio Pizzocchero**[2]

[1] Dipartimento di Matematica, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milano, Italy

[2] Dipartimento di Matematica, Università di Milano, Via C. Saldini 50, I-20133 Milano, Italy, and Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Italy

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Abstract: We give a Lie superalgebraic interpretation of the biHamiltonian structure of known supersymmetric KdV equations. We show that the loop algebra of a Lie superalgebra carries a natural Poisson pencil, and we subsequently deduce the biHamiltonian structure of the supersymmetric KdV hierarchies by applying to loop superalgebras an appropriate reduction technique. This construction can be regarded as a superextension of the Drinfeld–Sokolov method for building a KdV-type hierarchy from a simple Lie algebra.

1. Introduction

The possibility of superextensions of the integrable evolution equations has been considered in the literature in the last ten years. In particular, the construction of super Korteweg–de Vries hierarchies has been extensively discussed (an exhaustive bibliography can be found in [IK]).

Here, we are interested only in supersymmetric theories (from now on denoted by sKdV), which are invariant under space supersymmetries; so, we will confine our attention to the hierarchies introduced in [MR, Mat, LM], and we will not consider the superequations proposed in [Kup, GO].

Recently, the biHamiltonian structures and the recursion operators for the known sKdV hierarchies have been determined. This remarkable result, due to [OP] and, independently, to [FMR], has been obtained by applying R-matrix theory to the algebra of super pseudo-differential operators in a suitable super-derivation. In this framework, it is essential that each one of the proposed sKdV hierarchies admits a Lax formulation.

In this paper, we are discussing the biHamiltonian structure of sKdV hierarchies from a different point of view, which is based on the theory of simple Lie superalgebras. The differences and the existing relations between the R-matrix approach and the Lie algebraic approach are better understood if they are preliminarily discussed in the framework of classical integrable systems.

For definiteness, let us consider the ordinary KdV equation $q_t = -q_{xxx} + 6qq_x$, where the range of the space variable x is assumed to be the one-dimensional