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Quantum Continual Measurements and a Posteriori Collapse on CCR

V. P. Belavkin *

Department of Mathematics, University of Nottingham, University Park, Nottingham NG7 2RD, England

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Abstract. A quantum stochastic model for the Markovian dynamics of an open system under the nondemolition unsharp observation which is continuous in time, is given. A stochastic equation for the posterior evolution of a quantum continuously observed system is derived and the spontaneous collapse (stochastically continuous reduction of the wave packet) is described. The quantum Langevin evolution equation is solved for the case of a quasi-free Hamiltonian in the initial CCR algebra with a linear output channel, and the posterior dynamics corresponding to an initial Gaussian state is found. It is shown for an example of the posterior dynamics of a quantum oscillator that any mixed state under a complete nondemolition measurement collapses exponentially to a pure Gaussian one.

Introduction

The time evolution of a quantum system under a sharp continuous in time observation cannot be described by any Schrödinger equation due to the stochastic irreversible nature of von Neumann reduction of the wave packet at any instant of measurement. An adequate model of the quantum unitary evolution giving a continuous collapse by a conditioning with respect to the measurements can be obtained in the framework of quantum stochastic (QS) calculus [1], firstly introduced for output nondemolition processes in [2, 3] and recently developed in a quite general form in [4–6]. A stochastic wave equation for an observed quantum system derived in [6] by using the quantum filtering method [5], provides an explanation of pure quantum relaxation of an atom under a complete observation [7] (Zeno paradox) and a Watch-Dog effect [8] for the reduced wave function of a quantum particle under the continuous observation.

* On leave of Absence from M.I.E.M., B. Vusovski Street 3/12, Moscow 109028, Russia