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The Construction of the sh-Lie-Algebra of Closed Bosonic Strings

Hans-Werner Wiesbrock

Institut für Theorie der Elementarteilchen, Arnimalle 14, W-1000 Berlin 33, FRG

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Abstract. We present a construction of the closed string algebra in terms of Gaussian processes and crossed products. Also we give a purely functional analytical prove of the sh-Lie-structure.

1. Introduction

In 1989 M. Saadi and B. Zwiebach, [S–Z], inspired by the work of M. Kaku, [K], introduced an interaction of closed string fields along polyhedra. This was further investigated and generalized by T. Kugo, H. Kunimoto and K. Suehiro, [K–K–S, K–S]. They called it a nonpolynomial closed string field theory. They presented the theory in the operator formalism using the techniques of conformal theory and complex analysis. The nonpolynomial interaction of closed string fields should obey a gauge invariance, which was reformulated as an algebraic property. This later property was proved in [K–S] and for more general background fields in [S]. It was J. Stasheff who recognized that these algebraic relations define a strongly homotopy Lie algebra (sh-Lie-algebra), [St1]. The aim of this paper is to construct this algebra of closed string fields by using techniques of functional analysis. Also we will give an analytic proof of the sh-Lie-structure.

2. The Closed String Bosonic Fields

A classical closed string in the Euclidean formulation is given by a continuous imbedding

$$\omega: S^1 \to \mathbf{R}^d. \tag{1}$$

In the Schrödinger picture of quantization the closed string fields are functionals on such paths. The measure of integration is given by the free dynamics of the classical string.