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Even Dimensional Generalization of Chern-Simons Action and New Gauge Symmetry

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Abstract. We propose a new even-dimensional action which shares close algebraic similarities with the Chern-Simons action and thus possesses a topological nature. This action has a new type of gauge symmetry in the sense that adjoint representation is not enough to close the gauge transformation and gauge fermions are incorporated. "Quaternionic structure" emerges as a natural algebra to control the different natures of even forms, odd forms, bosons and fermions. We claim that the bundle structure in consideration is mathematically a new object.

Recently three-dimensional Chern-Simons action [1] played a fundamental role to understand two-dimensional conformal field theory [2] and three-dimensional gravity [3]. It has, however, been well known that Chern-Simons action is applicable only in three dimensions.

In this paper we show that it is possible to construct a new even-dimensional action which shares close algebraic similarities with the Chern-Simons action and possesses a new type of gauge symmetry. We claim that this construction defines mathematically a new "bundle structure" and topological invariants. It is obviously impossible to extend naively the three-dimensional Chern-Simons action into even dimensions. We need to introduce new ingredients and structures. In this paper we introduce antisymmetric tensors, gauge fermions and "quaternionic structure." There have been several trials of topological field theories in connection with Chern-Simons action and anti symmetric tensors [4, 5].

We first recall the procedure to prove the gauge invariance of the standard three-dimensional Chern-Simons action

$$S=\int \mathrm{Tr}(\tfrac{1}{2}\mathrm{Ad}A+\tfrac{1}{3}A^3), \tag{1}$$

where A is one form gauge field and carries non-Abelian gauge suffix. The above action is invariant under the following gauge transformation: $\delta A=dv+[A,v]$, where v is a zero form gauge parameter. We need the following two fundamental properties to prove the gauge invariance of the action: