Commun. Math. Phys. 138, 521–535 (1991)

Communications in
Mathematical
Physics

Markov Partition in Non-Hyperbolic Interval Dynamics*

Edson Vargas

Departamento de Matemática, PUC/RJ, 22453, Rio de Janeiro, RJ, Brazil

Received August 15, 1990; in revised form December 12, 1990

Abstract. We consider C^2 unimodal maps f such that all periodic points are hyperbolic, the critical point is non-degenerated and non-recurrent, and the Julia set does not contain intervals. We construct a Markov partition for a big part of the Julia set. Then we use it to estimate the limit capacity and Hausdorff dimension of the Julia set.

Introduction

Understanding the dynamical behavior of interval maps is a very interesting task. Besides having amazing and rich dynamics, they are part of more complicated dynamics. We mention some facts about the dynamics of a unimodal map: from the topological point of view we have a good description of the dynamics by the kneading theory, Milnor and Thurston [6]. Melo and Strien [5] and Martens et al. [4] generalized some theorems of structure and finiteness proved before, on the negative schwarzian case, by Guckenheimer [1] and by Singer [7], respectively. From the metrical point of view some problems are still open, for example: the size of invariant compact sets, the relation between stability and hyperbolicity, the existence of Bowen-Ruelle-Sinai measures, etc.

Here we analyze the Julia set Σ of unimodal Misiurewicz maps (non-recurrent critical point) with respect to its limit capacity and Hausdorff dimension. Hausdorff dimension is a nice way to measure the size of a set, how much it is dense. Limit capacity relies on geometrical properties of the set, how well it is distributed. Usually limit capacity is bigger than Hausdorff dimension. In the case of Σ they are equal. They are bigger than zero and in the case that Σ does not contain intervals, they are smaller than one. In fact, when Σ does not contain intervals, we can see a lot of order inside it. We can essentially realize it as a compact invariant set of a very nice Markov map.

The paper is divided into three sections. Section 1 contains precise statements and preliminary results. Section 2 is a key one, there we construct the "first hit"

* Partially supported by CNPq, S.C.T.-Brazil