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q-Weyl Group and a Multiplicative Formula for Universal R-Matrices*

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Abstract. We define the q-version of the Weyl group for quantized universal enveloping algebras of simple Lie group and we find explicit multiplicative formulas for the universal R-matrix.

1. For any semisimple complex Lie algebra $\mathscr{G}$ there is a natural deformation of its universal enveloping algebra $U\mathscr{G}$ as a Hopf algebra over the formal power series over $\mathbf{C}$ [D1, J]. This deformation $U_h\mathscr{G}$ is called a quantum universal enveloping algebra or quantum group [D1]. These algebras are important in the theory of quantum integrable systems [F] because with each $U_h\mathscr{G}$ one can associate a certain canonical element R in $(U_h\mathscr{G})^{\otimes 2}$ which satisfies the Yang-Baxter equation

$$R_{12}R_{13}R_{23}=R_{23}R_{13}R_{12}.$$

Here $R_{ij}\in U_h\mathscr{G}^{\otimes 3}$, $R_{12}=R\otimes 1$, $R_{23}=1\otimes R$ and $R_{13}=\sum_i \alpha_i\otimes 1\otimes\beta_i$ if we rewrite R as $R=\sum_i \alpha_i\otimes\beta_i$, α_i, $\beta_i\in U_h\mathscr{G}$.

But up to now there was no explicit formula for R, except for the cases $g=sl_2$ [D1], $\mathscr{G}=sl_n$ [Ro2]. Drinfeld (private communication) conjectured that there is a relation between the Weyl group and the universal R-matrix for general simple Lie algebras. In this paper we define a completion $U_h\mathscr{G}$ by the Weyl elements of sl_2 triples corresponding to simple roots. This completion gives us a description of the quantum Weyl group as well as explicit formulas for the element R.

2. Let $\mathscr{G}$ be a semisimple Lie algebra of rank n, a_{ij} its Cartan matrix, and d_i the length of the i-th root (then $d_ia_{ij}=a_{ji}d_j$).

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