Commun. Math. Phys. 121, 421–444 (1989)

Communications in Mathematical Physics

The Fermion Determinant in Two Dimensional Minkowski Space: Zeros and Related Properties*

D. Cangemi, M. Makowka and G. Wanders
Institut de Physique Théorique, Université de Lausanne, CH-1015 Lausanne, Switzerland**

Abstract. We present a detailed analysis of the non-abelian determinant of massless fermions in two dimensional Minkowski space. In the framework of the external field problem, the determinant vanishes if the out- and ingoing vacua are orthogonal; the gauge potentials for which this happens are identified. Causality implies that the effective action obtained from the sum of fermion loops has the right singularity at a zero of the determinant. Such a zero can be reached by a continuous deformation of a potential with non-vanishing determinant. The set of zeros exhibits a rich structure.

1. Introduction and Statement of Results

It has been realized some time ago that the determinant of massless fermion fields coupled to an external non-abelian gauge potential in a two dimensional space can be determined explicitly [1]. In a previous work [2] which will be referred to as I, we derived the effective action, that is the logarithm of the determinant, in the framework of $1+1$ dimensional Minkowski space-time.

At first sight our effective action, as well as the action obtained in a euclidean space, seems to be finite for all reasonable potentials. If this would be the case, the fermion determinant would never vanish. On the other hand, one knows that the determinant in a compactified euclidean space vanishes whenever the external potential allows zero modes of the fermion fields [3]. As nothing seems to prevent the occurrence of zero modes in two dimensions, there is a problem, at least in the euclidean setting, which has been addressed by Kupiainen and Mickelsson [4]. They find that there are potentials for which the fermion fields have zero modes and to which the known formula of the effective action does not apply. The main motivation of the present work was to find out if the Minkowski space determinant too has zeros and how they are compatible with our expression of the effective

* Work partially supported by the Swiss National Science Foundation

** Postal address: Bâtiment des Sciences physiques, Université de Lausanne, CH-1015 Lausanne, Switzerland