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Renormalizability and Infrared Finiteness of Non-Linear σ-Models: A Regularization-Independent Analysis for Compact Coset Spaces

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Abstract. The non-linear σ models in two space-time dimensions corresponding to compact homogeneous coset spaces G/H are studied with particular attention to three problems: first, independence of coordinate choice and regularization, second, the physical content of the theory, and finally the regularity of the "physics" in the infrared limit. Concerning in particular the physical content of the theory, we construct a set of local observables whose correlation functions depend on a finite number of parameters identified among those defining the metric tensor of the coset space. For these models, we give a general proof of renormalizability based on the introduction of a nilpotent BRS operator which describes the non-linear isometries and a classical action which contains a mass term for all quantized fields. The mass term belongs to a finite dimensional representation of the group G, which allows us to prove the conjecture that the correlation functions of local observables, i.e., the local operators invariant under G, are regular in the infrared limit.

1. Introduction

The non-linear σ-models were introduced more than 15 years ago [1, 2] to describe the infrared properties in $d>2$ space-time dimensions of systems with a symmetry spontaneously broken according to the Goldstone-Nambu mechanism. In 2 space-time dimensions – where the theory is power counting renormalizable – they appear as an interesting testing ground of theoretical ideas due to their asymptotic freedom property [3] and, more recently, because of their connection with the ground state of the string theories [4]. Consequently many efforts have been devoted to their investigation, both in perturbative and non-perturbative quantum field theory [5].

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