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Rotation Numbers of Periodic Orbits in the Hénon Map

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Abstract. For invertible, area-contracting maps of the plane, it is common for a basin of attraction to have a fractal basin boundary. Certain periodic orbits on the basin boundary are distinguished by being accessible (by a path) from the interior of the basin. A numerical study is made of the accessible periodic orbits for the Hénon family of maps. Theoretical results on rotary homoclinic tangencies are given, which describe the appearance of the accessible saddles, and organize them in a natural way according to the continued fractions expansions of their rotation numbers.

Introduction

Let f denote an area-contracting invertible map of the plane and let U be the basin of attraction of an attractor A. Although U may be easy to describe topologically (for example, U may be connected and simply connected, and thus homeomorphic to a disk), the boundary of U can be extremely complicated. In [7], it is pointed out that much of the basin boundary behavior can be characterized by certain distinguished orbits on the basin boundary, called accessible orbits. A point x is *accessible* from an open set U if there is a path, beginning in U, such that x is the first point not in U reached by the path. For real-valued maps, when the basin boundaries are fractal, most of the points in the basin boundary are not accessible.

When U is connected and simply connected, the map f acts on the accessible points as if they were on a circle. This fact, known to Birkhoff [4], means that one can associate a rotation number to the boundary points accessible from U. We call this number the *accessible rotation number*. It is shown in [2] that if the accessible rotation number is rational, then there exist accessible periodic orbits.

The goal of this paper is to study changes in the accessible rotation number as a map parameter is varied. The results of a computer investigation of a typical two-

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